

$$\frac{d(au)}{dx} = a \frac{du}{dx} \quad ; \quad \frac{du}{dt} = \frac{du}{dx} \cdot \frac{dx}{dt}$$

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} \quad ; \quad \frac{d(u/v)}{dx} = \frac{1}{v^2} \frac{du}{dx} - u \frac{dv}{dx}$$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$\frac{d}{dx}(\sin x) = \cos x \quad ; \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad ; \quad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x \quad ; \quad \frac{d}{dx}(\operatorname{cosec}^2 x) = -\cot x \operatorname{cosec} x$$

$$\frac{d}{dx}(u)^n = n u^{n-1} \frac{du}{dx} \quad ; \quad \frac{d}{du}(\ln u) = \frac{1}{u}$$

$$\frac{d}{du}(e^u) = e^u$$

In terms of derivatives, instantaneous velocity and acceleration are defined as

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Integral Calculus

You are familiar with the notion of area. The formulae for areas of simple geometrical figures are also known to you. For example, the area of a rectangle is length times breadth and that of a triangle is half of the product of base and height. But how to deal with the problem of determination of area of an irregular figure? The mathematical notion of integral is necessary in connection with such problems.

Let us take a concrete example. Suppose a variable force $f(x)$ acts on a particle in its motion along x -axis from $x = a$ to $x = b$. The problem is to determine the work done (W) by the force on the particle during the motion. This problem is discussed in detail in Chapter 6.

Figure 3.31 shows the variation of $F(x)$ with x . If the force were constant, work would be simply the area $F(b-a)$ as shown in Fig. 3.31(i). But in the general case, force is varying.

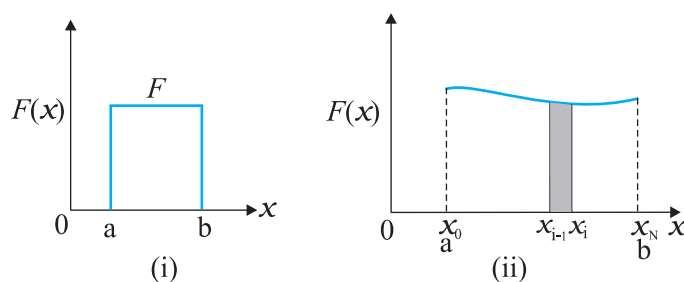


Fig. 3.31

To calculate the area under this curve [Fig. 3.31 (ii)], let us employ the following trick. Divide the interval on x -axis from a to b into a large number (N) of small intervals: $x_0(=a)$ to x_1 , x_1 to x_2 ; x_2 to x_3 , x_{N-1} to $x_N(=b)$. The area under the curve is thus divided into N strips. Each strip is approximately a rectangle, since the variation of $F(x)$ over a strip is negligible. The area of the i^{th} strip shown [Fig. 3.31(ii)] is then approximately

$$\Delta A_i = F(x_i)(x_i - x_{i-1}) = F(x_i)\Delta x$$

where Δx is the width of the strip which we have taken to be the same for all the strips. You may wonder whether we should put $F(x_{i-1})$ or the mean of $F(x_i)$ and $F(x_{i-1})$ in the above expression. If we take N to be very very large ($N \rightarrow \infty$), it does not really matter, since then the strip will be so thin that the difference between $F(x_i)$ and $F(x_{i-1})$ is vanishingly small. The total area under the curve then is:

$$A = \sum_{i=1}^N \Delta A_i = \sum_{i=1}^N F(x_i)\Delta x$$

The limit of this sum as $N \rightarrow \infty$ is known as the integral of $F(x)$ over x from a to b . It is given a special symbol as shown below:

$$A = \int_a^b F(x)dx$$

The integral sign \int looks like an elongated S, reminding us that it basically is the limit of the sum of an infinite number of terms.

A most significant mathematical fact is that integration is, in a sense, an inverse of differentiation.

Suppose we have a function $g(x)$ whose derivative is $f(x)$, i.e. $f(x) = \frac{dg(x)}{dx}$

The function $g(x)$ is known as the indefinite integral of $f(x)$ and is denoted as:

$$g(x) = \int f(x)dx$$

An integral with lower and upper limits is known as a definite integral. It is a number. Indefinite integral has no limits; it is a function.

A fundamental theorem of mathematics states that

$$\int_a^b f(x) dx = g(x) \Big|_a^b \equiv g(b) - g(a)$$

As an example, suppose $f(x) = x^2$ and we wish to determine the value of the definite integral from $x=1$ to $x=2$. The function $g(x)$ whose derivative is x^2 is $x^3/3$. Therefore,

$$\int_1^2 x^2 dx = \frac{x^3}{3} \Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Clearly, to evaluate definite integrals, we need to know the corresponding indefinite integrals. Some common indefinite integrals are

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$$

$$\int \left(\frac{1}{x}\right) dx = \ln x \quad (x > 0)$$

$$\int \sin x \, dx = -\cos x \quad \int \cos x \, dx = \sin x$$

$$\int e^x dx = e^x$$

This introduction to differential and integral calculus is not rigorous and is intended to convey to you the basic notions of calculus.

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