

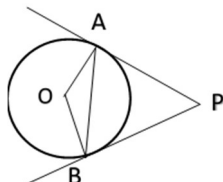
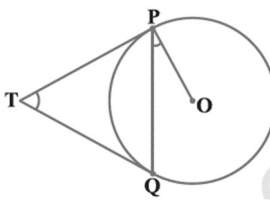
Marking Scheme
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours

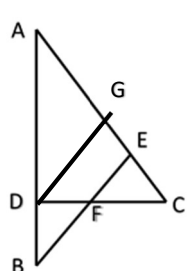
MAX.MARKS: 80

SECTION A		
Section A consists of 20 questions of 1 mark each.		
1.	(b) xy^2	1
2.	(b) 1 zero and the zero is '3'	1
3.	(b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	1
4.	(c) 2 distinct real roots	1
5.	(c) 7	1
6.	(a) 1:2	1
7.	(d) infinitely many	1
8.	(b) $\frac{ac}{b+c}$	1
9.	(b) 100°	1
10.	(d) 11 cm	1
11.	(c) $\frac{\sqrt{b^2-a^2}}{b}$	1
12.	(d) $\cos A$	1
13.	(a) 60°	1
14.	(a) 2 units	1
15.	(a) 10m	1
16.	(b) $\frac{4-\pi}{4}$	1
17.	(b) $\frac{22}{46}$	1
18.	(d) 150	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
SECTION B		
Section B consists of 5 questions of 2 marks each.		
21.	Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers a and b such that $\sqrt{2} = \frac{a}{b}$ where a and b are coprime. So, $b\sqrt{2} = a$. Squaring both sides, we get $2b^2 = a^2$. Therefore, 2 divides a^2 and so 2 divides a. So, we can write $a = 2c$ for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2, and so 2 divides b Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	2 (radius of the semi-circle) + side of a square = 8 cm $2a = 8 \text{ cm} \Rightarrow a = 4 \text{ cm}$ Area of the unshaded region = Area of a square of side 4 cm + 4 (Area of a semi-circle of diameter 4 cm) $= (4)^2 + 4 \times \frac{1}{2} \pi (2)^2 = (16 + 8\pi) \text{ cm}^2$	$\frac{1}{2}$ $\frac{1}{2}$
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Number of students in each group subject to the given condition = HCF (60,84,108) HCF (60,84,108) = 12 Number of groups in Music = $\frac{60}{12} = 5$ Number of groups in Dance = $\frac{84}{12} = 7$ Number of groups in Handicrafts = $\frac{108}{12} = 9$ Total number of rooms required = 21	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
27.	$P(x) = 5x^2 + 5x + 1$ $\alpha + \beta = \frac{-b}{a} = \frac{-5}{5} = -1$ $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= (-1)^2 - 2\left(\frac{1}{5}\right)$ $= 1 - \frac{2}{5} = \frac{3}{5}$ $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ $= \frac{(\alpha + \beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
28.	Let the ten's and the unit's digits in the first number be x and y, respectively. So, the original number = $10x + y$ When the digits are reversed, x becomes the unit's digit and y becomes the ten's Digit. So the obtain by reversing the digits = $10y + x$ According to the given condition. $(10x + y) + (10y + x) = 66$ i.e., $11(x + y) = 66$ i.e., $x + y = 6$ ---- (1) We are also given that the digits differ by 2, therefore, either $x - y = 2$ ---- (2) or $y - x = 2$ ---- (3) If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$. In this case, we get the number 42. If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	[or]	
	Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n', Then the given equations become $2m + 3n = 2$ $4m - 9n = -1$	$\frac{1}{2}$ $\frac{1}{2}$

	$\begin{aligned} (2m + 3n = 2) \times 2 &\Rightarrow -4m - 6n = -4 && \dots(1) \\ 4m - 9n &= -1 && 4m - 9n = -1 && \dots(2) \\ \text{Adding (1) and (2)} &&& && \\ \text{We get } -15n &= -5 \Rightarrow n = \frac{1}{3} \end{aligned}$	1/2
	Substituting $n = \frac{1}{3}$ in $2m + 3n = 2$, we get $\begin{aligned} 2m + 1 &= 2 \\ 2m &= 1 \\ m &= \frac{1}{2} \end{aligned}$	1/2
	$m = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4 \text{ and } n = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$	1
29.	$\angle OAB = 30^\circ$ $\angle OAP = 90^\circ$ [Angle between the tangent and the radius at the point of contact] $\angle PAB = 90^\circ - 30^\circ = 60^\circ$ $AP = BP$ [Tangents to a circle from an external point] $\angle PAB = \angle PBA$ [Angles opposite to equal sides of a triangle] In $\triangle ABP$, $\angle PAB + \angle PBA + \angle APB = 180^\circ$ [Angle Sum Property] $60^\circ + 60^\circ + \angle APB = 180^\circ$ $\angle APB = 60^\circ$ $\therefore \triangle ABP$ is an equilateral triangle, where $AP = BP = AB$. $PA = 6 \text{ cm}$ In Right $\triangle OAP$, $\angle OPA = 30^\circ$ $\tan 30^\circ = \frac{OA}{PA}$ $\frac{1}{\sqrt{3}} = \frac{OA}{6}$ $OA = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$ 	1/2 1/2 1/2 1/2 1/2 1/2
	[or]	
	Let $\angle TPQ = \theta$ $\angle TPO = 90^\circ$ [Angle between the tangent and the radius at the point of contact] $\angle OPQ = 90^\circ - \theta$ $TP = TQ$ [Tangents to a circle from an external point] $\angle TPQ = \angle TQP = \theta$ [Angles opposite to equal sides of a triangle] In $\triangle PQT$, $\angle PQT + \angle QPT + \angle PTQ = 180^\circ$ [Angle Sum Property] $\theta + \theta + \angle PTQ = 180^\circ$ $\angle PTQ = 180^\circ - 2\theta$ $\angle PTQ = 2(90^\circ - \theta)$ $\angle PTQ = 2 \angle OPQ$ [using (1)] 	1/2 1/2 1/2 1/2 1/2 1/2
30.	Given, $1 + \sin^2 \theta = 3 \sin \theta \cos \theta$ Dividing both sides by $\cos^2 \theta$, $\frac{1}{\cos^2 \theta} + \tan^2 \theta = 3 \tan \theta$ $\sec^2 \theta + \tan^2 \theta = 3 \tan \theta$ $1 + \tan^2 \theta + \tan^2 \theta = 3 \tan \theta$ $1 + 2 \tan^2 \theta = 3 \tan \theta$ $2 \tan^2 \theta - 3 \tan \theta + 1 = 0$ If $\tan \theta = x$, then the equation becomes $2x^2 - 3x + 1 = 0$	1/2 1/2 1/2 1/2

	$\Rightarrow (x - 1)(2x - 1) = 0 \text{ } x = 1 \text{ or } \frac{1}{2}$ $\tan \theta = 1 \text{ or } \frac{1}{2}$						1																																																
31.	<table><tr><th>Length [in mm]</th><th>Number of leaves (f)</th><th>CI</th><th>Mid x</th><th>d</th><th>fd</th></tr><tr><td>118 – 126</td><td>3</td><td>117.5- 126.5</td><td>122</td><td>-27</td><td>-81</td></tr><tr><td>127 – 135</td><td>5</td><td>126.5– 135.5</td><td>131</td><td>-18</td><td>-90</td></tr><tr><td>136 – 144</td><td>9</td><td>135.5– 144.5</td><td>140</td><td>-9</td><td>-81</td></tr><tr><td>145 – 153</td><td>12</td><td>144.5 – 153.5</td><td>a = 149</td><td>0</td><td>0</td></tr><tr><td>154 – 162</td><td>5</td><td>153.5 – 162.5</td><td>158</td><td>9</td><td>45</td></tr><tr><td>163 – 171</td><td>4</td><td>162.5 – 171.5</td><td>167</td><td>18</td><td>72</td></tr><tr><td>172 – 180</td><td>2</td><td>171.5 – 180.5</td><td>176</td><td>27</td><td>54</td></tr></table> $\text{Mean} = a + \frac{\sum fd}{\sum f} = 149 + \frac{-8}{40}$ $= 149 - 2.025 = 146.975$ Average length of the leaves = 146.975						Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd	118 – 126	3	117.5- 126.5	122	-27	-81	127 – 135	5	126.5– 135.5	131	-18	-90	136 – 144	9	135.5– 144.5	140	-9	-81	145 – 153	12	144.5 – 153.5	a = 149	0	0	154 – 162	5	153.5 – 162.5	158	9	45	163 – 171	4	162.5 – 171.5	167	18	72	172 – 180	2	171.5 – 180.5	176	27	54	2 $\frac{1}{2}$ $\frac{1}{2}$
Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd																																																		
118 – 126	3	117.5- 126.5	122	-27	-81																																																		
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163 – 171	4	162.5 – 171.5	167	18	72																																																		
172 – 180	2	171.5 – 180.5	176	27	54																																																		
	SECTION D																																																						
	Section D consists of 4 questions of 5 marks each																																																						
32.	Let the speed of the stream be x km/h. The speed of the boat upstream = (18 – x) km/h and the speed of the boat downstream = (18 + x) km/h. The time taken to go upstream = $\frac{\text{distance}}{\text{speed}} = \frac{24}{18-x}$ hours the time taken to go downstream = $\frac{\text{distance}}{\text{spe}} = \frac{24}{18+x}$ hours According to the question, $\frac{24}{18-x} - \frac{24}{18+x} = 1$ $24(18 + x) - 24(18 - x) = (18 - x)(18 + x)$ $x^2 + 48x - 324 = 0$ $x = 6 \text{ or } -54$ Since x is the speed of the stream, it cannot be negative. Therefore, x = 6 gives the speed of the stream = 6 km/h.						1 																																																

	Therefore, $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$ $8x^2 - 230x + 750 = 0$ $x = 25, \frac{30}{8}$ Time taken by the smaller pipe cannot be $30/8 = 3.75$ hours, as the time taken by the larger pipe will become negative, which is logically not possible. Therefore, the time taken individually by the smaller pipe is 25 hours and the larger pipe will be $25 - 10 = 15$ hours.	1/2 1 1/2 1/2
33.	(a) Statement – 1/2 Given and To Prove – 1/2 Figure and Construction 1/2 Proof – 1 1/2 [b] Draw $DG \parallel BE$ In $\triangle ABE$, $\frac{AB}{BD} = \frac{AE}{GE}$ [BPT] $CF = FD$ [F is the midpoint of DC] ---(i) In $\triangle CDG$, $\frac{DF}{CF} = \frac{GE}{CE} = 1$ [Mid point theorem] $GE = CE$ ---(ii) $\angle CEF = \angle CFE$ [Given] $CF = CE$ [Sides opposite to equal angles] ---(iii) From (ii) & (iii) $CF = GE$ ---(iv) From (i) & (iv) $GE = FD$ $\therefore \frac{AB}{BD} = \frac{AE}{GE} \Rightarrow \frac{AB}{BD} = \frac{AE}{FD}$ 	3 1/2 1/2 1/2 1/2
34.	Length of the pond, $l = 50\text{m}$, width of the pond, $b = 44\text{m}$ Water level is to rise by, $h = 21\text{ cm} = \frac{21}{100}\text{ m}$ Volume of water in the pond $= lbh = 50 \times 44 \times \frac{21}{100}\text{ m}^3 = 462\text{ m}^3$ Diameter of the pipe $= 14\text{ cm}$ Radius of the pipe, $r = 7\text{cm} = \frac{7}{100}\text{ m}$ Area of cross-section of pipe $= \pi r^2$ $= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000}\text{ m}^2$ Rate at which the water is flowing through the pipe, $h = 15\text{km/h} = 15000\text{ m/h}$ Volume of water flowing in 1 hour $= \text{Area of cross-section of pipe} \times \text{height of water coming out of pipe}$ $= \left(\frac{154}{10000} \times 15000 \right) \text{ m}^3$ Time required to fill the pond $= \frac{\text{Volume of the pond}}{\text{Volume of water flowing in 1 hour}}$ $= \frac{462 \times 10000}{154 \times 15000} = 2\text{ hours}$ Speed of water if the rise in water level is to be attained in 1 hour $= 30\text{km/h}$ [or]	1 1/2 1 1

	$= 40 + \frac{25-15}{2(25)-15-20} \cdot 10$ $= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$	
	SECTION E	
36.	(i) Number of throws during camp. $a = 40$; $d = 12$ $t_{11} = a + 10d$ $= 40 + 10 \times 12$ $= 160 \text{ throws}$	1
	(ii) $a = 7.56$ m; $d = 9\text{cm} = 0.09$ m $n = 6 \text{ weeks}$ $t_n = a + (n-1) d$ $= 7.56 + 6(0.09)$ $= 7.56 + 0.54$ Sanjitha's throw distance at the end of 6 weeks = 8.1 m (or) $a = 7.56 \text{ m; } d = 9\text{cm} = 0.09 \text{ m}$ $t_n = 11.16 \text{ m}$ $t_n = a + (n-1) d$ $11.16 = 7.56 + (n-1) (0.09)$ $3.6 = (n-1) (0.09)$ $n-1 = \frac{3.6}{0.09} = 40$ $n = 41$ Sanjitha's will be able to throw 11.16 m in 41 weeks.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	(iii) $a = 40$; $d = 12$; $n = 15$ $S_n = \frac{n}{2} [2a + (n-1) d]$ $S_n = \frac{15}{2} [2(40) + (15-1) (12)]$ $= \frac{15}{2} [80 + 168]$ $= \frac{15}{2} [248] = 1860 \text{ throws}$	$\frac{1}{2}$ $\frac{1}{2}$
37.	(i) Let D be (a,b), then Mid point of AC = Midpoint of BD $\left(\frac{1+6}{2}, \frac{2+6}{2} \right) = \left(\frac{4+a}{2}, \frac{3+b}{2} \right)$ $4 + a = 7 \quad 3 + b = 8$ $a = 3 \quad b = 5$ Central midfielder is at (3,5)	$\frac{1}{2}$ $\frac{1}{2}$

	(ii) $GH = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ $GK = \sqrt{(0+3)^2 + (3-5)^2} = \sqrt{9+4} = \sqrt{13}$ $HK = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13}$ $GK + HK = GH \Rightarrow G, H \text{ \& K lie on a same straight line}$ [or] $CJ = \sqrt{(0-5)^2 + (1+3)^2} = \sqrt{25+16} = \sqrt{41}$ $CI = \sqrt{(0+4)^2 + (1-6)^2} = \sqrt{16+25} = \sqrt{41}$ Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) Mid-point of IJ = $\left(\frac{5-4}{2}, \frac{-3+6}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$ C is NOT the mid-point of IJ	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	(iii) A,B and E lie on the same straight line and B is equidistant from A and E $\Rightarrow B$ is the mid-point of AE $\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2, -3)$ $1+a = 4; a = 3. \quad 4+b = -6; b = -10$ E is (3,-10)	$\frac{1}{2}$ $\frac{1}{2}$
38.	(i) $\tan 45^\circ = \frac{80}{CB} \Rightarrow CB = 80\text{m}$	1
	(ii) $\tan 30^\circ = \frac{80}{CE}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{CE}$ $\Rightarrow CE = 80\sqrt{3}$ Distance the bird flew = AD = BE = CE-CB = $80\sqrt{3} - 80 = 80(\sqrt{3}-1)$ m (or) $\tan 60^\circ = \frac{80}{CG}$ $\Rightarrow \sqrt{3} = \frac{80}{CG}$ $\Rightarrow CG = \frac{80}{\sqrt{3}}$ Distance the ball travelled after hitting the tree = FA=GB = CB -CG $GB = 80 - \frac{80}{\sqrt{3}} = 80\left(1 - \frac{1}{\sqrt{3}}\right)$ m	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	(iii) Speed of the bird = $\frac{\text{Distance}}{\text{Time taken}} = \frac{20(\sqrt{3}+1)}{2}$ m/sec $= \frac{20(\sqrt{3}+1)}{2} \times 60 \text{ m/min} = 600(\sqrt{3}+1) \text{ m/min}$	$\frac{1}{2}$ $\frac{1}{2}$